

PROBLEM SET 2
MATH 411: HONORS COMPLEX VARIABLES, FALL 2026

Suggested completion date: Sep 28, 2026

- (1) Give the terms of order ≤ 3 in the power series (using the expressions for $\cos(T)$, $\sin(T)$, and e^T given in lecture):

(a) $\frac{\sin T}{\cos T}$;

(b) $\frac{e^T}{\sin T}$.

- (2) Given complex numbers a_0, a_1, u_1, u_2 , define a_n for $n \geq 2$ by

$$a_n = u_1 a_{n-1} + u_2 a_{n-2}.$$

If

$$T^2 - u_1 T - u_2 = (T - \alpha)(T - \alpha')$$

with $\alpha \neq \alpha'$, show that

$$a_n = A\alpha^n + B(\alpha')^n$$

for suitable A, B . Find A, B in terms of $a_0, a_1, \alpha, \alpha'$. Consider the power series

$$F(T) = \sum_{n=0}^{\infty} a_n T^n.$$

Show that it represents a rational function $f(T)/g(T)$, where $f(T)$ and $g(T)$ are polynomials with $g(T) \neq 0$. Give the partial fraction decomposition of this rational function.

- (3) Determine the radius of convergence of the following series:

(a) $\sum n^2 z^n$;

(b) $\sum \frac{n!}{n} z^n$;

- (c) If $f = \sum a_n z^n$ has radius of convergence $r > 0$, show that $\sum n^d a_n z^n$ has the same radius of convergence for any positive integer d .

- (4) Prove the following:

- (a) The power series $\sum n z^n$ has radius of convergence 1 but does not converge at any point of the unit circle.

- (b) The power series $\sum \frac{z^n}{n^2}$ has radius of convergence 1 and converges at every point of the unit circle.

- (c) The power series $\sum \frac{z^n}{n}$ converges at every point of the unit circle except $z = 1$.

- (5) Find the terms of order ≤ 3 in the power series expansion of

$$f(z) = \frac{z-2}{(z+3)(z+2)}$$

at $z = 1$.