

Midterm 1 - Class results

30 exams | Raw scores out of 32 | No scaling applied

EXAMS

30

MEAN

20.60 / 32

MEDIAN

21.25 / 32

POPULATION SD

6.43 points

MINIMUM

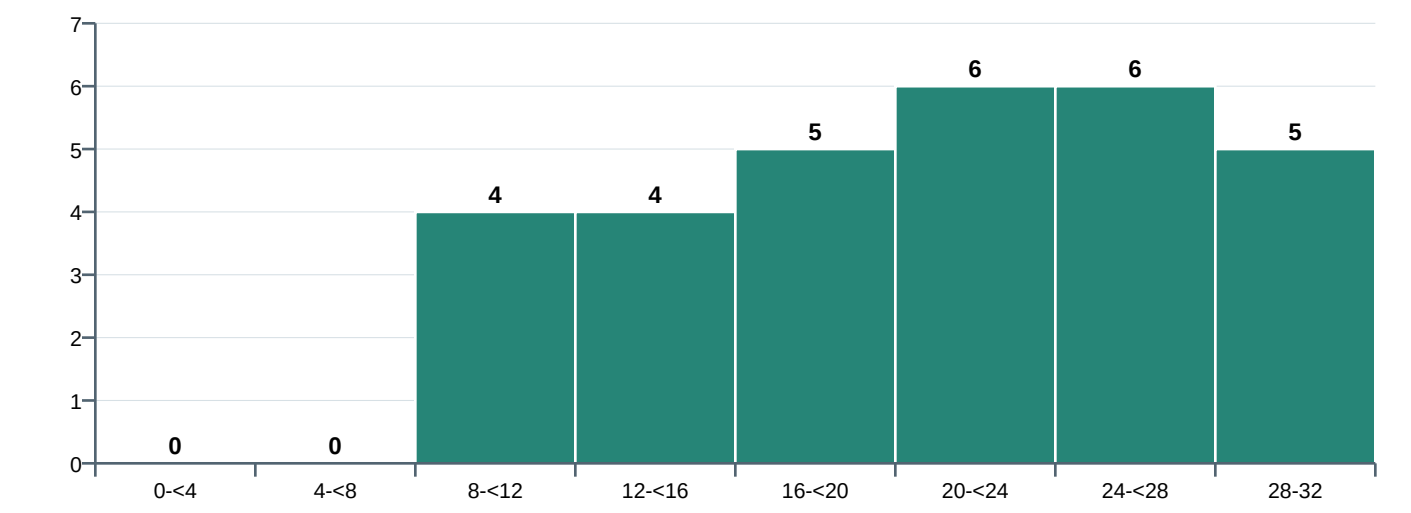
8 / 32

MAXIMUM

29.5 / 32

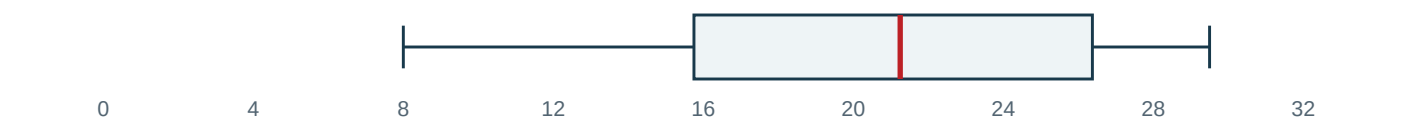
Score histogram

Four-point bins; the final bin includes 32. Labels show exam counts.



Score / 32

Spread of scores



Q1: 15.75 | Median: 21.25 | Q3: 26.375 | IQR: 10.625 points

Whiskers show minimum and maximum; quartiles use inclusive linear interpolation.

Section	Class mean / 8	Mean percent
Short questions	6.58	82.3%
Question 1	4.68	58.5%
Question 2	5.05	63.1%
Question 3	4.28	53.5%

Question performance

Class averages | All 30 exams | Raw marks

Part	Maximum	Class mean	Mean percent	Full credit
A1	2	1.60	80.0%	23 / 30
A2	2	1.78	89.2%	23 / 30
B1	1	0.77	76.7%	23 / 30
B2	1	0.77	76.7%	23 / 30
C1	1	0.90	90.0%	27 / 30
C2	1	0.77	76.7%	23 / 30
1(a)	2	1.60	80.0%	19 / 30
1(b)	3	1.48	49.4%	5 / 30
1(c)	3	1.60	53.3%	12 / 30
2(a)	3	2.28	76.1%	17 / 30
2(b)	2	1.15	57.5%	13 / 30
2(c)	3	1.62	53.9%	12 / 30
3(a)	2	1.45	72.5%	16 / 30
3(b)	2	1.23	61.7%	9 / 30
3(c)	4	1.60	40.0%	0 / 30

How to read the expanded marking guidance

The detailed rubric on pages 3-6 explains the mathematical targets, point allocations recorded in the marking policy, and examples of partial-credit awards. Page 7 summarizes class trends.

This is a retrospective explanation of the existing grading. Where the ledger did not prescribe a fixed component split, the rubric reports observed score levels rather than inventing a new split. Individual feedback records the marks awarded; all class statistics include the current set of graded exams.

Sources and scope

Based on midterm1-grading-ledger.json: 30 exams, item-level marks, marking policy, and written feedback; checked against the Midterm 1 instructor solutions. All percentages refer to raw scores. No scaling or letter-grade conversion is applied.

Grading rubric - Short questions

8 points | A1-A2: 2 each | B1-C2: 1 each

General marking principles

Marks total 32, with 8 in each section. Blank answers receive 0. Substantive correct work earns partial credit, usually in 0.5-point steps. Equivalent valid arguments and notation are accepted. Short-answer and objective items are assessed on the requested answers; no extra proof requirement is introduced here.

A1. Order and generated subgroup - 2 points

1 point: the order of 6 modulo 18 is 3. **1 point:** the generated subgroup is $\{0, 6, 12\}$ modulo 18. The operation is addition; the subgroup consists of multiples of 6, reduced modulo 18.

Recorded awards: one correct component earned 1/2; both incorrect earned 0/2. Equivalent residue representatives, such as -6 for 12, were accepted. Listing both 0 and 18 was treated as redundant, without a deduction when the intended subgroup was clear.

A2. Kernel and image - 2 points

1 point: $\ker(\phi) = 8\mathbb{Z}$ in the domain $\mathbb{Z}$. **1 point:** $\text{im}(\phi) = \mathbb{Z}/8\mathbb{Z}$, including the zero residue class.

Recorded awards: a correct component plus a partly specified other component could earn 1.5/2, for example an image list missing zero. One correct component earned 1/2. Correct defining conditions without evaluating either set also earned 1/2. The kernel is all multiples of 8, rather than just the identity or a residue class.

B1. Equality of left cosets - 1 point

1 point: choice (b), $b^{-1}a$ belongs to H . **0 points:** an incorrect choice. Equal cosets can have different representatives; $a = b$ is too strong.

B2. Subgroups of the integers - 1 point

1 point: choice (d), the statement that $a < b$ implies $a\mathbb{Z}$ contains $b\mathbb{Z}$ is not always true. **0 points:** another choice. For example, $2 < 3$ but 3 is not in $2\mathbb{Z}$. Numerical inequality alone does not imply divisibility or subgroup inclusion.

C1. Subgroups of an abelian group - 1 point

1 point: True. Every subgroup of an abelian group is normal, since $ghg^{-1} = h$. **0 points:** False.

C2. All real 2 by 2 matrices - 1 point

1 point: False. Singular matrices, including the zero matrix, lack multiplicative inverses. **0 points:** True.

Grading rubric - Question 1

8 points | Permutations and commutation

1(a). Compose permutations in disjoint cycles - 2 points

Full credit: (2 5 3 4) using rightmost-first composition, or (1 4 3 5) using leftmost-first composition. The exam did not specify a convention, so either consistent convention receives 2/2. Cycle rotations and omission of fixed points are equivalent forms.

Recorded partial credit: 1.5/2 for the correct permutation written in factors that were not disjoint; 1/2 for useful mapping work with an incorrect or unfinished cycle answer, including conflicting final answers; 0.5/2 for limited composition work with a substantially incorrect result.

Why points were lost: a correct element-by-element mapping must be converted into the requested cycle form. A product such as (2 5)(3 4) is a different permutation from (2 5 3 4). A redundant one-cycle overlapping the main cycle does not satisfy the disjoint-cycle requirement.

1(b). Order and parity of an odd-length cycle - 3 points

1 point: order k . **1 point:** even parity. **1 point:** valid justification, for example a k -cycle is a product of $k - 1$ transpositions, so its sign is $(-1)^{k-1} = 1$ when k is odd.

Recorded partial credit: 2/3 for correct order and parity without a valid general parity argument, or for correct parity with justification but incorrect order. An almost complete explanation earned 2.5/3. A correct order alone, or correct parity alone, earned 1/3. Recognition of the order definition without the requested values earned 0.5/3 in one response.

Why points were lost: cycle length, permutation order, and number of transpositions are different quantities. The order is k ; a standard decomposition uses $k - 1$ transpositions. An example with a three-cycle alone does not prove the statement for every odd k .

1(c). Transfer commutation from a cubed to a - 3 points

Full credit: use $\text{ord}(a) = 7$ and $a^3b = ba^3$ to prove $ab = ba$. One route is $a = (a^3)^5$, since 15 is 1 modulo 7; powers of a^3 commute with b . Another route establishes commutation with a^4 , then squares to get $a^8 = a$.

Recorded score levels: 2/3 for establishing commutation with a^4 but omitting the final step; 1/3 for useful, correct rearrangements without reaching the conclusion; 0.5/3 for limited valid identities; 0/3 for no valid progress, a blank, or a circular argument.

Why points were lost: factors cannot be reordered or cancelled across opposite sides without justification. Conjugate elements need not be equal, and $\text{ord}(a) = 7$ does not imply $(ab)^7 = e$ or that G is abelian.

Grading rubric - Question 2

8 points | Subgroups and normality

2(a). The intersection of two subgroups - 3 points

1 point: nonemptiness, using e in both H and K . **1 point:** product closure, proving that for x, y in $H \cap K$, xy lies in both subgroups. **1 point:** inverse closure, proving x^{-1} lies in both subgroups.

Equivalent proof: after establishing nonemptiness, show xy^{-1} lies in H and in K , hence in their intersection. A valid one-step subgroup test receives the same total credit; separate product and inverse arguments are unnecessary.

Recorded partial credit: 2/3 for the identity and inverse closure with product closure missing; 1.5/3 for a correct identity argument with incomplete closure work; 1/3 for a common identity or recognition of the subgroup test without the required closure proof. Special cases alone earned limited credit (0.5/3). Nonempty sets can have empty intersection, so nonemptiness must be established using the common identity.

2(b). A union that is not a subgroup - 2 points

1 point: specify a valid ambient group and two actual subgroups. **1 point:** demonstrate that the union fails to be a subgroup. For example, $G = (\mathbb{Z}, +)$, $H = 2\mathbb{Z}$, $K = 3\mathbb{Z}$; 2 and 3 lie in the union but $2 + 3 = 5$ does not.

Other valid arguments: concrete examples in $\mathbb{Z}/6\mathbb{Z}$ or S_3 were accepted. A correct S_3 example whose union has four elements can also be ruled out by Lagrange, since 4 does not divide 6.

Recorded partial credit: 1/2 for a valid setup without a correct nonclosure demonstration, or for a useful witness with incorrect subgroup or ambient-group notation; 1.5/2 for a recognizable valid example with a smaller presentation gap; 0.5/2 for a promising but incomplete construction. An invalid setup with no usable demonstration, or no example, earned 0/2. $\mathbb{Z}/2\mathbb{Z}$ and $2\mathbb{Z}$ are different objects; the proposed subgroups must actually lie in the stated ambient group.

2(c). Normality of a homomorphism kernel - 3 points

Up to 1 point: sound kernel/subgroup groundwork; the known fact that the kernel is a subgroup may be used. **Up to 2 points:** a valid conjugation argument. For arbitrary g in G and k in $\ker(\phi)$, compute $\phi(gkg^{-1}) = \phi(g)\phi(k)\phi(g)^{-1} = e$, and conclude gkg^{-1} belongs to $\ker(\phi)$.

Recorded partial credit: 1/3 for subgroup groundwork or correct kernel/normality ideas without establishing conjugation invariance; 0.5/3 for limited relevant work, such as conjugating only the identity; 0/3 for a blank or no substantive valid argument.

Why points were lost: k in the kernel means $\phi(k) = e$ in the target, not $k = e$ in the domain. Equal images do not imply equal elements unless the map is injective. A valid general conjugation-inclusion proof received full credit.

Grading rubric - Question 3

8 points | Element orders and counting

Throughout Question 3, $p < q$ are primes and $|G| = pq$. The exam requires a proof without Cauchy's theorem or a result directly implying the desired existence statement.

3(a). Possible orders of a nonidentity element - 2 points

Full credit: apply Lagrange to the cyclic subgroup generated by x , obtaining $\text{ord}(x)$ dividing pq ; list the divisors 1, p , q , pq ; exclude 1 because x is not the identity. Therefore the possibilities are p , q , and pq .

Recorded partial credit: 1.5/2 for a substantially correct argument with a missing explicit link, typically the exclusion of order 1 or the invocation of Lagrange; 1/2 for a correct major step without completion; 0.5/2 for a relevant order or prime-factor idea without the divisibility proof; 0/2 for a blank or no valid progress. The feedback assesses the whole argument rather than prescribing a finer component split.

3(b). Produce order p from order pq - 2 points

Full credit: given $\text{ord}(x) = pq$, choose $y = x^q$ and establish its exact order p . A direct proof shows $y^p = e$ and y is not e (otherwise $\text{ord}(x)$ would divide q). Since p is prime, the order is p . The formula $\text{ord}(x^q) = pq/\gcd(pq, q) = p$ is also sufficient.

Recorded partial credit: 1.5/2 for the correct power and an incomplete exact-order argument, usually the omitted nonidentity check; 1/2 for inconsistent candidate powers with the order asserted rather than proved; 0/2 for no valid construction, an unsupported existence claim, or a blank.

Why points were lost: $y^p = e$ only proves the order divides p . Also exclude order 1. Knowing that p divides $|G|$ does not by itself prove the required existence under the permitted methods.

3(c). Count order- q subgroups and obtain a contradiction - 4 points

Up to 2 points - intersections: under the assumption that every nonidentity element has order q , prove distinct order- q subgroups intersect only in the identity. Their intersection has order dividing q ; an intersection of order q would make the subgroups equal. Equivalent common-generator arguments are valid.

Up to 1 point - counting: let N count distinct order- q subgroups. Every nonidentity element generates one, and each contributes $q - 1$ distinct nonidentity elements. Thus $pq - 1 = N(q - 1)$. Count the identity once; count subgroups, rather than one new subgroup for each generator.

Up to 1 point - contradiction and conclusion: $pq - 1 = p(q - 1) + (p - 1)$, with $0 < p - 1 < q - 1$. Hence $q - 1$ cannot divide $pq - 1$. Some nonidentity element therefore has order p or pq by (a); apply (b) in the latter case.

Recorded awards: correct intersections and count with an unsupported final contradiction earned 3/4; an almost explicit final step earned 3.5/4. Incomplete combinations earned 0.5-2.5/4 according to the valid progress recorded. For example, a correct count with an incomplete common-generator explanation and an invalid final contradiction earned 2.5/4. Blank answers earned 0/4. No response received 4/4.

Trends and recurring errors

Separate analysis of the recorded scores and feedback | 30 exams

The class mean was **20.60/32 (64.4%)** and the median was **21.25/32 (66.4%)**. Scores ranged from 8 to 29.5, with a population standard deviation of 6.43 points. The middle half lay between 15.75 and 26.375. These results show substantial variation in performance.

Section	Mean / 8	Mean percent
Short questions	6.58	82.3%
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1. Short answers were stronger than longer proofs

The short-question mean was 82.3%, compared with 58.5%, 63.1%, and 53.5% on Questions 1-3. C1 was strongest (27/30 correct); A1 and A2 earned full credit on 23 and 23 exams. Feedback on incorrect answers nevertheless showed confusion between a residue class, a generated subgroup, a kernel in the domain, and an image in the codomain.

2. Familiar computations did not ensure conceptual justification

Permutation composition in 1(a) averaged 80.0%, with 19/30 full-credit answers. Cycle order and parity in 1(b) averaged 49.4%, with only 5/30 full-credit answers. Comments repeatedly distinguished cycle length, order, and transposition count. In 1(c), 12/30 completed the commutation proof; recurring gaps included invalid reordering, cancellation, and stopping before transferring commutation to a itself.

3. Applying definitions was harder than recognizing them

The intersection proof in 2(a) averaged 76.1%, with 17/30 full-credit answers. The union counterexample in 2(b) averaged 57.5%, with 13/30 full-credit answers; common problems were invalid ambient groups, incorrect subgroup notation, and missing nonclosure witnesses. In 2(c), 18/30 earned at most 1/3, while 12/30 earned full credit. Repeated errors replaced a kernel element by the domain identity or inferred equality from equal images.

4. Exact-order and counting proofs often stopped one step early

In 3(b), 12/30 earned 1.5/2; the comments consistently called for a nonidentity check or completion of the exact-order argument. Question 3(c) was the weakest item (40.0%): 0 full-credit answers, although 7/30 earned at least 3/4. Overcounting generators, counting q instead of $q - 1$ nonidentity elements, and unsupported divisibility contradictions were recurring obstacles.