

Extra practice for Midterm

MATH 327: Algebra 1

Fall 2026

Midterm 1

The first three exercises are from Michael Artin's *Algebra*, second edition, Chapter 2, Section 8 (Cosets).

Exercise 8.5

A finite group G contains an element x of order 10 and also an element y of order 6. What can be said about the order of G ?

Exercise 8.7

A group G of order 22 contains elements x and y , where $x \neq 1$ and y is not a power of x . Prove that the subgroup generated by these elements is the whole group G .

Exercise 8.8

Let G be a group of order 25. Prove that G has at least one subgroup of order 5, and that if it contains only one subgroup of order 5, then it is a cyclic group.

Short-answer practice

Give the requested answers; no proof is required. Compose permutations rightmost first.

Exercise S1

In the additive group $\mathbb{Z}/20\mathbb{Z}$, find the order of $\bar{8}$ and list the elements of $\langle \bar{8} \rangle$.

Exercise S2

Let $\varphi : \mathbb{Z} \rightarrow \mathbb{Z}/12\mathbb{Z}$ be the homomorphism $\varphi(n) = \overline{4n}$. Describe $\ker \varphi$ and list the elements of $\operatorname{im} \varphi$.

Exercise S3

Find the positive integers d and e such that

$$8\mathbb{Z} + 12\mathbb{Z} = d\mathbb{Z}, \quad 8\mathbb{Z} \cap 12\mathbb{Z} = e\mathbb{Z}.$$

Exercise S4

In the additive group $\mathbb{Z}/12\mathbb{Z}$, let $H = \langle \bar{4} \rangle$. List the elements of the coset $\bar{1} + H$. Is $\bar{9} + H$ the same coset?

Short-answer practice (continued)

Exercise S5

Let $\sigma = (1\ 5\ 2\ 4)(3\ 6) \in S_6$. Find σ^{-1} , the order of σ , and $\text{sgn}(\sigma)$.

Exercise S6

Let G be a group of order 42 and $H \leq G$ a subgroup of order 7. How many left cosets of H are there? Can G contain an element of order 8?

Exercise S7

Let $\psi : G \rightarrow K$ be a homomorphism, where $|G| = 24$ and $|\ker \psi| = 6$. Find $|\text{im } \psi|$. If $|K| = 8$, is ψ surjective?

Exercise S8

Let $H = \{1, (1\ 3)\} \leq S_3$. List the elements of the left coset $(1\ 2)H$ and of the right coset $H(1\ 2)$. Are these cosets equal?

True or false

Mark each statement true or false. For review, also try to give a brief reason or a counterexample.

T1. Every cyclic group is abelian.

T **F**

T2. Every abelian group is cyclic.

T **F**

T3. Every left coset of a subgroup $H \leq G$ is a subgroup of G .

T **F**

T4. If H is a finite subgroup of G , then every left coset of H has $|H|$ elements.

T **F**

T5. If $\varphi : G \rightarrow K$ is a homomorphism and $a \in G$ has finite order, then $\varphi(a)$ has the same order as a .

T **F**

T6. A group homomorphism with trivial kernel is injective.

T **F**

T7. If $a \in G$ has finite order and $g \in G$, then gag^{-1} has the same order as a .

T **F**

T8. The product of two odd permutations is odd.

T **F**

T9. The set of invertible 2×2 real matrices is a group under matrix multiplication.

T **F**

T10. $4\mathbb{Z} \subseteq 6\mathbb{Z}$.

T **F**