

MATH 327: Algebra 1

Midterm 1

Fall 2026 | Friday, October 9, 2026

Instructor: Manish Patnaik

Time: 50 minutes Total: 32 points

Student name: _____

Student ID: _____

Directions

1. This is a closed-book exam. No notes, calculators, or electronic devices may be used.
2. Answer all questions. Check that your exam has seven pages: this cover, one page of short questions, three pages of written-response questions, and two extra-work pages.
3. Write clearly in the spaces provided. Pages 6 and 7 may be used for extra work. If you continue an answer there, write “Continued on page 6” or “Continued on page 7” in its original answer space. Label the extra work with the question and part number.
4. For the short-answer questions, give the requested answer; no proof is required. For multiple choice, circle exactly one answer. For true/false, circle T or F; no justification is required.
5. For Questions 1–3, show your work and justify your claims. Partial credit may be awarded for meaningful progress.
6. You may use results proved in class unless a question asks you to prove that result. State clearly any theorem you use, and check its hypotheses. Do not use results from later topics that have not been covered.

For instructor use only

Section	Points available	Score
Short questions (page 2)	8	
Question 1 (page 3)	8	
Question 2 (page 4)	8	
Question 3 (page 5)	8	
Total	32	

Short questions**[8 points]****A. Short answer (2 points each)**

A1. In the additive group $\mathbb{Z}/18\mathbb{Z}$, find the order of $\bar{6}$ and list the elements of $\langle \bar{6} \rangle$.

A2. Let $\varphi : \mathbb{Z} \rightarrow \mathbb{Z}/8\mathbb{Z}$ be given by $\varphi(n) = \bar{n}$. Describe $\ker \varphi$ and $\operatorname{im} \varphi$.

B. Multiple choice (1 point each)

B1. Let $H \leq G$ and $a, b \in G$. Which condition is equivalent to $aH = bH$?

- | | |
|------------------------|---------------------|
| (a) $ab^{-1} \in H$ | (b) $b^{-1}a \in H$ |
| (c) $a^{-1}b \notin H$ | (d) $a = b$ |

B2. Let $a, b, c \in \mathbb{Z}$ be positive integers. Which of the following statements is **NOT** always true?

- (a) $\gcd(a, b) = ma + nb$ for some integers $m, n \in \mathbb{Z}$.
- (b) $a\mathbb{Z} + b\mathbb{Z} + c\mathbb{Z}$ is a cyclic subgroup of $\mathbb{Z}$.
- (c) If $\gcd(a, b) = 1$, then $a\mathbb{Z} + b\mathbb{Z} = \mathbb{Z}$.
- (d) If $a < b$, then $a\mathbb{Z} \supseteq b\mathbb{Z}$.

C. True or false (1 point each)

C1. Every subgroup of an abelian group is normal.

T F

C2. The set $M_2(\mathbb{R})$ of all 2×2 real matrices is a group under matrix multiplication.

T F

Question 1**[8 points]**

(a) **[2 points]** Let $p = (1\ 4\ 2)(3\ 5)$ and $q = (1\ 2\ 3)$ in S_5 . Compute pq and express it as a product of **disjoint cycles**.

(b) **[3 points]** Let $\sigma \in S_n$ be a cycle $\sigma = (i_1\ i_2\ \cdots\ i_k)$ with k odd. What is the **order** of σ , and is σ an **even or an odd** permutation? Justify briefly.

(c) **[3 points]** Let a and b be elements of a group G . Suppose a has order 7 and $a^3b = ba^3$. Show that

$$ab = ba.$$

Question 2**[8 points]**

Let H and K be subgroups of a group G .

- (a) **[3 points]** Prove that $H \cap K$ is a subgroup of G .
- (b) **[2 points]** Give an example of a group G and subgroups $H, K \leq G$ for which $H \cup K$ is not a subgroup of G . Specify G , H , and K explicitly.
- (c) **[3 points]** Let $\varphi : G \rightarrow G'$ be a homomorphism of groups. Show that $\ker(\varphi)$ is a normal subgroup of G .

Question 3**[8 points]**

Let p and q be prime numbers with $p < q$, and let G be a group of order pq . Show that G has an element of order p .

Do not use Cauchy's theorem or another theorem that directly implies the result.

(a) **[2 points]** Let $x \in G$ with $x \neq 1$. Show that $\text{ord}(x) \in \{p, q, pq\}$.

(b) **[2 points]** If some element has order pq , show that G has an element of order p .

(c) **[4 points]** Suppose every non-identity element of G has order q . Show that two distinct subgroups of order q intersect only in $\{1\}$, and count elements to reach a contradiction. Conclude that G has an element of order p .

Extra work – page 6

Label each continuation with its question and part number.

Question and part: _____

Extra work – page 7

Label each continuation with its question and part number.

Question and part: _____